Charge Detection in a Closed-Loop Aharonov-Bohm Interferometer

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(Dated: February 1, 2008)

We report on a study of complementarity in a two-terminal *closed-loop* Aharonov-Bohm interferometer. In this interferometer, the simple picture of two-path interference cannot be applied. We introduce a nearby quantum point contact to detect the electron in a quantum dot inserted in the interferometer. We found that charge detection reduces but does not completely suppress the interference even in the limit of perfect detection. We attribute this phenomenon to the unique nature of the closed-loop interferometer. That is, the closed-loop interferometer cannot be simply regarded as a two-path interferometer because of multiple reflections of electrons. As a result, there exist indistinguishable paths of the electron in the interferometer and the interference survives even in the limit of perfect charge detection. This implies that charge detection is not equivalent to path detection in a closed-loop interferometer. We also discuss the phase rigidity of the transmission probability for a two-terminal conductor in the presence of a detector.

Complementarity in quantum theory is well described in a two-path interferometer such as Young's double slit interferometer. In a two-path interferometer with a 'which-path' detector, observation of the interference pattern and the acquisition of which-path information are mutually exclusive [1, 2, 3]. Most of the work on understanding this kind of interferometer has been carried out in optical systems with photons [4]. Only recently has it become possible to investigate the complementarity of electrons in solid-state circuits [5, 6, 7]. The interference is shown by the oscillation of conductance as a function of magnetic flux in an Aharonov-Bohm (AB) interferometer with a quantum dot (QD) inserted in one of its arms. This AB oscillation of conductance has been observed both in a closed [8] and in an open-geometry [9]. The open-geometry AB interferometer of Ref. 9 can be regarded as a solid-state version of Young's double slit interferometer. A mesoscopic which-path interferometer has been demonstrated by using an open-geometry AB interferometer containing a quantum dot (QD) with a nearby quantum point contact (QPC) used as a which-path detector [5]. The QPC interacts with the QD and is able to detect a single charge in the QD. The detection is made through the QD-charge dependence of the scattering coefficients at the QPC. This results in decoherence of the charge state of the QD and suppression of the AB oscillation. The suppression strength is controlled through the voltage across the QPC. Different setups for the controlled dephasing experiment have been also demonstrated by using QD-QPC hybrid structures [6, 7].

It is obvious that charge detection is equivalent to the path detection in a double-slit (or two-path) AB interferometer investigated in Ref. 5. On the other hand, it is an interesting question as to what would happen in a closed-loop AB interferometer of the type studied in Ref. 8

when a charge detector is attached. In a closed-loop AB interferometer, the conductance through the system is not simply given by interference between electron transmission through the two direct paths [10]. Therefore, a charge detection may not be equivalent to the path detection in this closed-loop interferometer.

In this Letter, we report on our investigation of complementarity in a closed-loop AB interferometer, where the simple picture of Young's double slit is invalid. A QD is embedded in one arm of the interferometer, and is coupled to a QPC being used as a charge detector (See Fig. 1). In contrast to that of a two-path interferometer, we show that the AB oscillation is not completely suppressed even in the limit of perfect charge detection. This feature originates from the fact that charge detection does not entirely determine the path of an electron in a closed-loop interferometer.

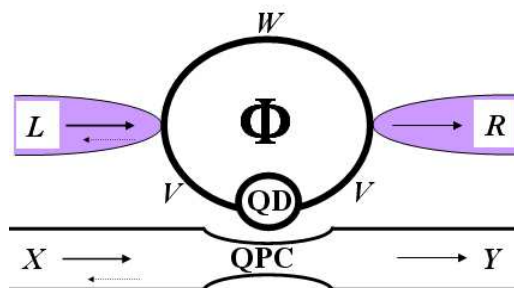The Hamiltonian of a closed-loop AB interferometer

FIG. 1: Schematic figure of the model system: A closed-loop Aharonov-Bohm interferometer with a quantum point contact detector. A quantum dot is inserted in the interferometer and coupled to the quantum point contact which enables detection of the charge state in the quantum dot.

with a QD inserted in one of its arms is given by $H_{AB} = H_0 + H_1$, where

$$H_0 = \epsilon_d d^\dagger d + \sum_\alpha \sum_k \epsilon_k c^\dagger_{\alpha k} c_{\alpha k} , \qquad (1a)$$

$$H_1 = \sum_k [(W c^\dagger_{Lk} c_{Rk} + V d^\dagger c_{Lk} + V c^\dagger_{Rk} d) + h.c.] . (1b)$$

H_0 denotes a QD and the two leads (L, R). H_1 describes transfer of electrons between the subsystems. The operator d $(d^\dagger)$ annihilates (creates) an electron in the QD with energy ϵ_d. The operator $c^\dagger_{\alpha k}$ and $c_{\alpha k}$ $(\alpha = L, R)$ refer to states in the lead α. The hoping amplitudes V and W are chosen as $V = |V|$ and $W = |W| e^{i\varphi}$ with φ being the AB phase.

A nearby QPC detector close to the QD is introduced. Because of the Coulomb interactions between the QD and the QPC, the electron state in the QPC depends on the trajectory of electron in the interferometer. Accordingly, dephasing of the QD electron state takes place. Various theoretical approaches have been reported that address this issue [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. In a two-path which-path interferometer, the visibility reduction factor of the AB oscillation is proportional to the dephasing rate. Naturally, AB oscillation in a two-path interferometer disappears in the perfect detection limit in which the two detector states are orthogonal.

The situation is different for a closed-loop interferometer. Quantum interference originates from the indistinguishability of two or more events. In fact, there exist *indistinguishable paths* of the electron in the closed-loop interferometer despite perfect charge detection. This is because charge detection does not provide complete information on the electron's path. Examples are shown in Fig. 2. Considering an electron injected from the left lead which is detected by the QPC at the QD (Fig. 2(a)), one cannot determine whether this electron came from the left (full arrow) or from the right (dashed arrow) arm. Similarly, for an electron detected at the QD and then absorbed in the right lead, one cannot determine a definite path as shown in Fig. 2(b). This kind of indistinguishability does not exist in a double-slit type interferometer
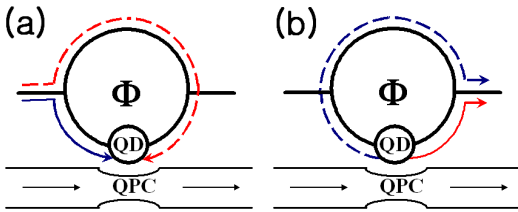

FIG. 2: Examples of indistinguishable paths for an electron transport with charge detection at the QD. (a) Two indistinguishable paths from the left lead to the QD. (b) Two indistinguishable paths from the QD to the right lead.

where a charge detection is equivalent to path detection. In our setup, these kind of events contribute to the AB oscillation of the conductance through the interferometer in the perfect charge detection limit.

Let us now describe quantitatively the hybrid system of a closed-loop interferometer and a QPC detector. For simplicity, we assume that the two electrons, one from lead L of the interferometer and the other from lead X of the detector (See Fig. 1), are simultaneously injected. Also, our discussion is restricted to the off-resonance limit of the QD. Then, upon a scattering of the two electrons, we can write the two-particle state as

$$|\psi\rangle \simeq |\phi_0\rangle_e \otimes |\chi_0\rangle_d + |\phi_1\rangle_e \otimes |\chi_1\rangle_d , \qquad (2)$$

where $|\phi_0\rangle_e$ denotes all possible paths which do not go into the QD. The state $|\phi_1\rangle_e$ includes all processes that includes the leading (second) order tunneling through the QD. Higher order tunneling processes can be neglected in the off-resonance limit. Note that $|\phi_0\rangle_e$ and $|\phi_1\rangle_e$ include multiple reflections at the contacts between the leads and the interferometer. These multiple reflections make the system different from a two-path interferometer. $|\chi_0\rangle_d$ and $|\chi_1\rangle_d$ represent the corresponding detector states. These states can be written as

$$|\chi_i\rangle_d = \bar{r}_i |X\rangle + \bar{t}_i |Y\rangle \ \ (i \in 0, 1), \qquad (3)$$

where $\bar{r}_i$ and $\bar{t}_i$ are the i-dependent reflection and transmission amplitudes, respectively. $|X\rangle$ and $|Y\rangle$ are the states of the electron being at lead X and Y, respectively.

For the state $|\psi\rangle$ of Eq.(2), the probability of finding an electron at lead R (equivalent to the transmission probability) is given as

$$T_{LR} = \int \left(-\frac{\partial f}{\partial \epsilon} \right) \langle \psi | R \rangle \langle R | \otimes I_d | \psi \rangle d\epsilon , \qquad (4)$$

where f is the Fermi distribution function. $|R\rangle$ corresponds to the state of the electron being at lead R. I_d is the identity operator that acts only on the detector. At zero temperature, one finds that

$$T_{LR} = |t_0|^2 + |t_1|^2 + 2\text{Re}[\lambda t_0^* t_1] , \qquad (5a)$$

where $t_0 = \langle R | \phi_0 \rangle_e$ and $t_1 = \langle R | \phi_1 \rangle_e$ are the transmission amplitudes for the state $|\phi_0\rangle_e$ and $|\phi_1\rangle_e$, respectively. The constant λ given as

$$\lambda = {}_d\langle \chi_0 | \chi_1 \rangle_d = \bar{r}_0^* \bar{r}_1 + \bar{t}_0^* \bar{t}_1 \qquad (5b)$$

is a measure of the indistinguishability between the states, $|\phi_0\rangle_e$ and $|\phi_1\rangle_e$.

In general, transmission amplitude, t_i $(i \in 0, 1)$, (in the absence of a 'detector') can be obtained from Green's function G_i $(i \in 0, 1)$ using the relation [22],

$$t_i = i\hbar\nu G_i , \qquad (5c)$$

where ν is the Fermi velocity. In our case, the Green's functions, G_i, (and the transmission coefficients t_i) which correspond to the states $|\phi_i\rangle_e$ are calculated by using perturbation expansion on the hopping part of the Hamiltonian (H_1). Diagrams for the infinite series of this perturbation expansion are given in Fig. 3. After some algebra, one finds that

$$t_0 = \frac{-2ix}{1+x^2}e^{-i\varphi} , \qquad (5d)$$

where $x = \pi\rho|W|$ with ρ being the density of states at the Fermi energy, and

$$t_1 = \frac{\Gamma}{\epsilon_d}t_0\left(2i - \frac{e^{i\varphi}}{x} + xe^{-i\varphi}\right) , \qquad (5e)$$

where Γ is the effective resonance width of the QD level given as $\Gamma = \pi\rho|V|^2/(1+x^2)$.

Fig. 4(a) shows the AB oscillations of the transmission probability. As one can see, the amplitude of the AB oscillation is reduced as λ decreases but does not vanish even in the limit of $\lambda = 0$. The visibility $\mathcal{V}$ defined as $\mathcal{V} = (\max(T_{LR}) - \min(T_{LR}))/(\max(T_{LR}) + \min(T_{LR}))$ is shown in Fig. 4(b) as a function of λ. The visibility is, in general, reduced as $|\lambda|$ decreases. However, the oscillation is not entirely suppressed even in the limit of perfect charge detection ($\lambda = 0$), in contrast to that of a 'double-slit' type interferometer. Note that the remaining visibility in the $\lambda \to 0$ limit comes from the fact that $|t_1|^2$ depends on the AB phase φ. In fact, the AB interference of $|t_1|^2$ results from the indistinguishability of various paths of the electron as shown in Fig. 3. This

is the origin of the finite visibility for $\lambda = 0$ where the interference between the states $|\phi_0\rangle_e$ and $|\phi_1\rangle_e$ vanishes.

One might interpret the visibility reduction through charge detection in the following way: Coulomb interaction between the electrons in the QD and in the detector disturbs the motion of an electron in the QD. This disturbing then results in an uncertainty of the electron's phase. This 'random phase' washes out the interference. In this interpretation, however, the closed-loop interferometer would not be different from the two-path interferometer. That is, any electron would loose its phase coherence whenever it passes through the QD, and the visibility of the AB oscillation vanishes in the strong detection limit. Then, the closed-loop interferometer would not show any essential difference from the double-slit interferometer, in spite of nontrivial electronic paths. Our results (Eq. 5 and Fig. 4) clearly indicate that the 'random-phase' interpretation does not apply to the charge detection in a closed-loop interferometer.

About the phase rigidity: It is well known that the transmission probability for a two-terminal conductor (without a detector) should satisfy the relation $T_{LR}(-\varphi) = T_{LR}(\varphi)$, so that a 'phase rigidity' exists [10].

FIG. 4: (a) The transmission probability through the closed-loop interferometer as a function of the AB Phase, and (b) the visibility of interference pattern (full line) as a function of λ, for $x = 0.4$, $V = 0.75|W|$, $\epsilon_d = 1.25|W|$, and $\Gamma/\epsilon_d = 0.155$. For comparison, the visibility of 'double-slit' interferometer (dashed line) is also shown for the same parameters.
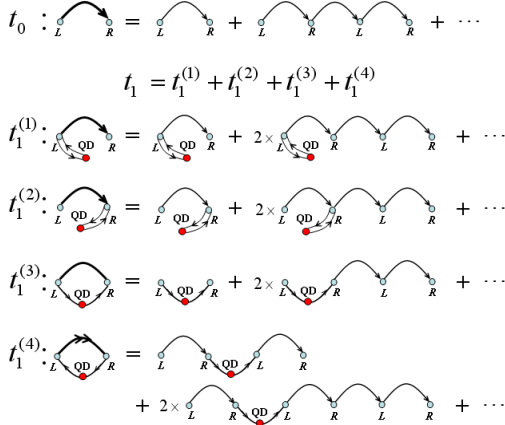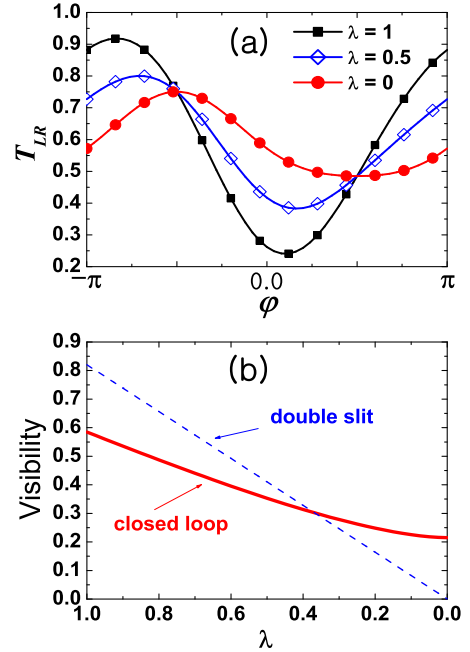

FIG. 3: Diagrams which represents the partial waves of an electron moving from lead L to R. t_0 represents the electron transition through the free arm. t_1 includes all possible diagrams with a single tunneling event through the QD. These can be classified into four different contributions, namely $t_1^{(1)}$, $t_1^{(2)}$, $t_1^{(3)}$, and $t_1^{(4)}$ as shown in the figure.

This phase rigidity is not satisfied in our result as one can see in Fig. 4 (a). One reason for the breaking of the phase rigidity is because of our approximations that neglect higher order tunneling processes. These approximations do not fully take into account the unitarity of electron scattering. Therefore, it naturally breaks the phase rigidity. However, there is a more fundamental reason for the phase rigidity breaking. That is, the phase rigidity does not exist in a system interacting with a detector. In the following, we develop a general argument on this lack of phase rigidity.

Let us consider two particle injection: one from lead L of the interferometer, and the other from lead X of the detector. The two electrons interact with each other at the QD-QPC contact region. Upon a scattering, the two electrons have four possible configurations, namely $|1\rangle \equiv |L\rangle \otimes |X\rangle$, $|2\rangle \equiv |L\rangle \otimes |Y\rangle$, $|3\rangle \equiv |R\rangle \otimes |X\rangle$, and $|4\rangle \equiv |R\rangle \otimes |Y\rangle$. With this representation, the initial state can be written as $|\psi_{in}\rangle = |1\rangle$. After scattering, the two-particle state is given by

$$|\psi(\varphi)\rangle = \hat{S}(\varphi)|\psi_{in}\rangle = \sum_{i=1}^{4} S_{i1}(\varphi)|i\rangle \,, \qquad (6)$$

where $\hat{S}$ denotes the two-particle scattering matrix and $S_{ij} \equiv \langle i|\hat{S}|j\rangle$. Note that $\hat{S}$ cannot be written as a direct product of two single-particle scattering matrices because of the interaction between the two subsystems. The reciprocity relation of $\hat{S}$ gives the constraint $S_{ij}(\varphi) = S_{ji}(-\varphi)$. Also, the unitarity of $\hat{S}$ requires $\sum_{i=1}^{4} |S_{i1}(\varphi)|^2 = \sum_{i=1}^{4} |S_{1i}(\varphi)|^2 = 1$. The transmission probability in the interferometer is given by

$$T_{LR}(\varphi) = \langle \psi(\varphi)|R\rangle\langle R|\otimes I_d|\psi(\varphi)\rangle = |S_{31}(\varphi)|^2 + |S_{41}(\varphi)|^2 \,. \qquad (7)$$

From Eq.(7) together with the unitarity and the reciprocity relations, we find that the phase rigidity $T_{LR}(-\varphi) = T_{LR}(\varphi)$ is satisfied if $|S_{21}(\varphi)|^2 = |S_{12}(\varphi)|^2$. However, this condition is not valid in general in the presence of two-particle interactions. (That is, $S_{21} = S_{12}$ only when the two systems are independent.) Therefore, we conclude that *the phase rigidity of a two-terminal conductor is not enforced, in general, if the conductor is interacting with another system.*

In conclusion, we studied the influence of charge detection on the conductance oscillation in a two-terminal *closed-loop* AB interferometer where the simple picture of a double-slit interference cannot be applied. We found that charge detection reduces but does not fully suppress the interference even in the limit of perfect detection. This interesting property originates from multiple reflections of an electron in the two-terminal interferometer. Because of the multiple reflections of an electron in the interferometer, full information about the electronic path cannot be obtained through charge detection. This *indistinguishability* of electronic paths results in the AB oscillation. Furthermore, based on a general argument, we pointed out that the phase rigidity is not enforced in a two-terminal conductor if the conductor is interacting with another subsystem.

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